QUIZ 5 SOLUTIONS: LESSON 3
JANUARY 18, 2019

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [5 pts] Compute $\displaystyle\int \frac{1}{2x(\ln(3x))^3}\,dx.$

Chain rule

$$\int \frac{1}{2x(\ln(3x))^3}\,dx = \int \frac{1}{2x}(\ln(3x))^{-3}\,dx$$

$$\boxed{\begin{array}{l} u = \ln(3x) \\ \frac{du}{dx} = \frac{3}{3x} = \frac{1}{x} \\ \Rightarrow dx = x\,du \end{array}}$$

$$= \int \frac{1}{2x} u^{-3}(x\,du)$$

$$= \int \frac{1}{2} u^{-3}\,du$$

$$= \frac{1}{2}\left(\frac{1}{-3+1}\right) u^{-3+1} + C$$

$$= \frac{1}{2}\left(\frac{-1}{2}\right) u^{-2} + C$$

$$= -\frac{1}{4} u^{-2} + C$$

$$\boxed{= -\frac{1}{4}\left(\ln(3x)\right)^{-2} + C}$$

2. [5 pts] Compute $\displaystyle\int_e^{e^2} \frac{1}{x\ln(x^{20})}\,dx.$

$$\int_e^{e^2} \frac{1}{x\ln(x^{20})}\,dx = \int_e^{e^2} \frac{1}{20x\ln x}\,dx$$

$$\boxed{\begin{array}{l} \ln x^{20} = 20\ln x \\ u = \ln x \\ \frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x\,du \\ u(e^2) = \ln e^2 = 2 \\ u(e) = \ln e^1 = 1 \end{array}}$$

$$= \int_e^{e^2} \frac{1}{20x}(\ln x)^{-1}\,dx$$

$$= \int_{u(e)}^{u(e^2)} \frac{1}{20x} u^{-1}(x\,du)$$

$$= \int_1^2 \frac{1}{20} u^{-1}\,du$$

$$= \frac{1}{20}\ln|u|\Big|_1^2$$

$$= \frac{1}{20}\ln 2 - \frac{1}{20}\underbrace{\ln 1}_{0}$$

$$\boxed{= \frac{1}{20}\ln 2}$$